Bethe-Salpeter equations: mesons beyond the rainbow-ladder truncation

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Abstract We investigate masses of light mesons from a coupled system of Dyson–Schwinger (DSE) and Bethe–Salpeter equations (BSE), taking into account dominant non-Abelian, sub-leading Abelian, and dominant pion cloud contributions to the dressed quark-gluon vertex. The axial-vector Ward-Takahashi identity preserving Bethe-Salpeter kernel is constructed and the spectrum of light mesons calculated. Our model goes significantly beyond the rainbow-ladder. We find that sub-leading Abelian corrections are further dynamically suppressed, and that our results supersede early qualitative predictions from simple truncation schemes.

Key words Dyson-Schwinger equations, Bethe-Salpeter equations, light-mesons, beyond rainbow-ladder

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1 Introduction

Using the Dyson-Schwinger and Bethe-Salpeter equations, there have been successful descriptions of pseudoscalar and vector mesons [1, 2, 3, 4, 5] using the simple rainbow-ladder (RL) truncation. This is not unexpected since for the spectrum of light pseudoscalar mesons, it is the axial-vector Ward-Takahashi identity that governs the pattern of dynamical symmetry breaking. These RL investigations have also been recently extended to describe baryons [6, 7]; a status report is given in these proceedings [8].

Considerable effort to go beyond RL, using the prescription of [9, 10], has been made in a number of works [10, 11, 12, 13, 14]. These however considered sub-leading Abelian corrections rather than the dominant non-Abelian parts of the quark-gluon vertex [15]. In Ref. [16] we performed a first calculation of the leading non-Abelian diagrams in a coupled DSE/BSE approach. Here we include the sub-leading Abelian corrections together with the pion back-reaction [17, 18].

2 Dyson-Schwinger Equations

To solve the DSEs for the quark propagator and quark-gluon vertex, shown in Fig. 1(a) and Fig. 1(b) we must introduce a truncation scheme. The BSE must be consistently truncated, being careful to preserve chiral symmetry. This is expressed via the axial-vector Ward-Takahashi identity (axWTI) and ensures that pions are the pseudo-Goldstone bosons.

The simplest truncation that satisfies this criterion is that of RL whereby the full quark-gluon vertex is replaced by a bare vertex [2]. The axWTI preserving kernel in the BSE then corresponds to a single gluon exchange, re-summed to all orders thus providing the 'ladder'. These RL models effectively subsume additional vertex corrections from the Yang-Mills sector and unquenching via an effective coupling.

To make an extension to the rainbow-ladder truncation, we investigate the quark-gluon vertex and determine the impact of corrections beyond tree-level to our quarks and mesons [10, 11, 12, 13, 14, 19, 20]. Following the analysis of [15, 21] we approximate the full DSE Fig. 1(a) with the (nonperturbative) one-loop structure of Fig. 1(b). Here the first 'non-Abelian' loop-diagram in Fig. 1(b) subsumes the first two diagrams in the full DSE to first order in a skeleton expansion of the four-point functions. We neglect the two-loop diagram in the full DSE Fig. 1(a), which is justified for small and large momenta [15, 19]. The remaining 'Abelian' contributions are split into the non-resonant second loop-diagram in Fig. 1(b) and a third diagram containing effects due to hadron back-reactions. The details of including the pion-backreaction are contained within Refs. [17, 21, 22], see therein for details. This provides an extension of our previous truncation involving only the non-Abelian diagram [16].

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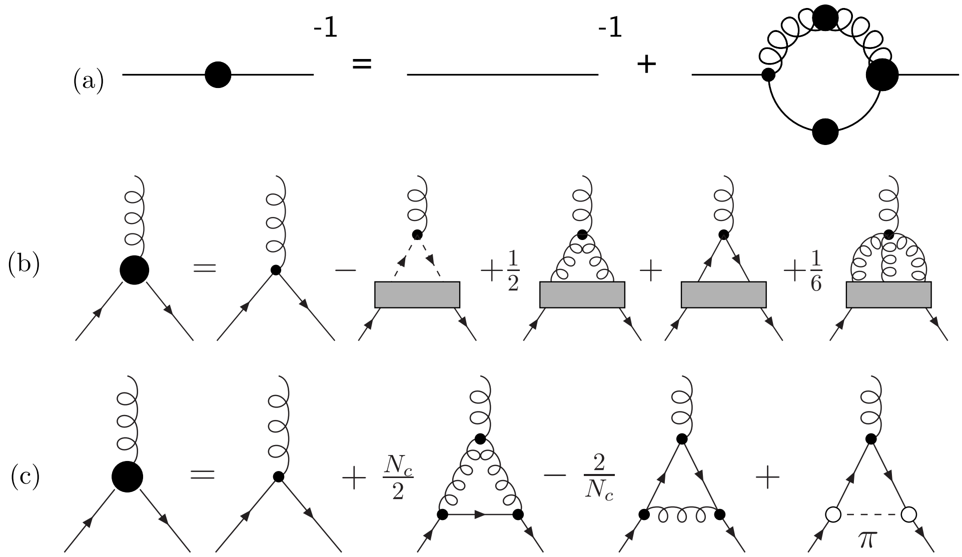

Fig. 1. DSEs for: (a) fully dressed quark propagator; (b) full quark-gluon vertex; (c) truncated quark-gluon vertex. Internal propagators are dressed, with gluons shown by wiggly lines, quarks by straight lines and dashed lines mesons. White-filled circles show meson amplitudes whilst black-filled represent vertex dressings.

3 Bethe-Salpeter equation

The Bethe-Salpeter equation describing a relativistic bound-state of mass M is calculated through

$$[\Gamma(p;P)]_{tu} = \lambda \int_k K^{rs}_{tu}(p,k;P)\left[S(k_+)\Gamma(k;P)S(k_-)\right]_{sr} . \quad (1)$$

Here $\int_k = \int \frac{d^4k}{(2\pi)^4}$, $\Gamma(p;P) \equiv \Gamma^{(\mu)}(p;P)$ is the Bethe-Salpeter vertex function of a quark-antiquark bound state. It is a homogeneous equation, with a discrete spectrum of solutions at momenta $P^2 = -M_i^2$ corresponding to $\lambda(P^2) = 1$. The lightest of these M_i pertains to the ground state solution. The momenta $k_+ = k{+}\eta P$ and $k_- = k{-}(1{-}\eta)P$ are such that the total momentum P of the meson is given by $P = k_+{-}k_-$ and the relative momentum $k = (k_+ + k_-)/2$. The momentum partitioning parameter is η, of which physical observables are independent. For convenience we choose it to be 1/2 without loss of generalisation. The object $K^{rs}_{tu}(p,k;P)$ is the Bethe-Salpeter kernel, whose Latin indices refer to colour, flavour and Dirac structure.

The Bethe-Salpeter vertex function $\Gamma^{(\mu)}(p;P)$ can be decomposed into eight Lorentz and Dirac structures. The structure is constrained by the transformation properties under CPT of the meson [23].

The consequences of chiral symmetry can be expressed through the axial-vector Ward-Takahashi identity for flavour non-singlet mesons

$$-iP_\mu\Gamma^5_\mu = S_F^{-1}(p_+)\gamma_5 + \gamma_5 S_F^{-1}(p_-) - 2m_R\Gamma^5(p;P) , \quad (2)$$

with p and P the relative and total momentum of the meson respectively, $p_\pm$ is combination $p{\pm}P/2$, Γ^5_μ the axial-vector and Γ^5 the pseudoscalar vertex.

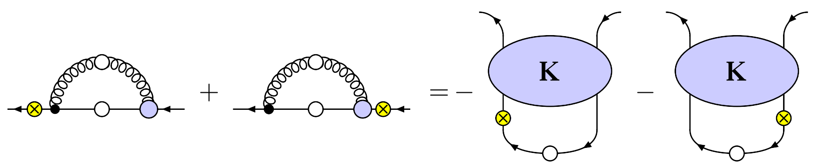

Fig. 2. The flavour non-singlet axWTI written showing the connection between the quark self-energy and Bethe-Salpeter kernel. All propagators are dressed, with wiggly and straight lines for gluons and quarks respectively. The crossed circle indicates a γ_5 insertion.

It is well-known that one may construct a Bethe-Salpeter kernel K^{rs}_{tu} satisfying the axWTI by means of a functional derivative of the quark self-energy. Applying this cutting procedure to the quark DSE of Fig. 1(a) with the quark-gluon vertex of Fig. 1(c) yields the Bethe-Salpeter equation portrayed in Fig. 3 [24]. Because we do not trivialize the momentum dependence of our propagators, the calculation is genuinely two-loop. Such a truncation allows us to consider the dominant non-Abelian corrections [16], together with the sub-leading Abelian corrections [13]. We are now in a position to perform a systematic comparison of the impact of each vertex correction on the spectrum of light mesons.

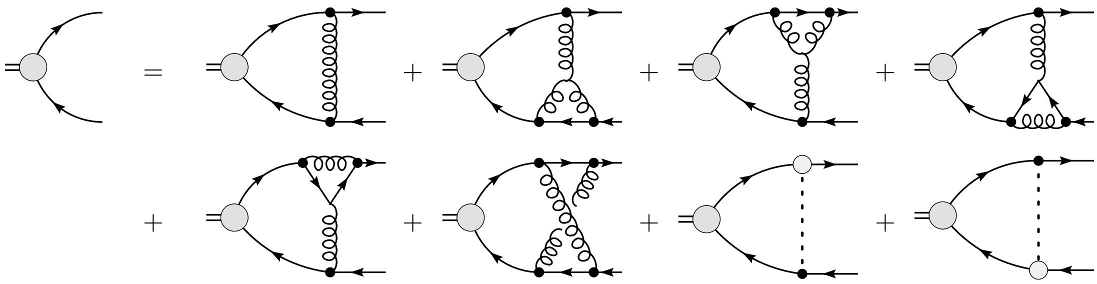

Fig. 3. The axWTI preserving BSE corresponding to our vertex truncation. All propagators are dressed, with wiggly and straight lines showing gluons and quarks respectively.

4 Results

Model	m_π	m_σ	m_ρ	m_{a_1}	m_{b_1}
RL	138	645	758	926	912
NA	142	884	881	1056	973
AB	137	602	734	889	915
AB+NA	142	883	878	1052	972
NA+PI	138	820	805	1040	941
PDG [25]	138	400–1200	776	1230	1230

Table 1. Masses for a variety of mesons calculated using rainbow-ladder (RL), additional corrections from the Abelian (AB) and non-Abelian (NA) diagrams, and with the pion backreaction (PI). Masses are given in MeV.

We present results in Table 1. The benchmark for comparison, for which the parameters of the interaction are fixed to meson observables, are those of the rainbow-ladder (RL) approximation. By including the dominant non-Abelian vertex correction (NA) we find only a small change in the pion mass, as expected since it is a pseudo-Goldstone boson. For the remaining bound states we typically see an enhancement of the mass of the order of 100–200 MeV, depending upon the meson channel. That is, inclusion of the non-Abelian diagram is repulsive in the meson channels considered here.

If we turn off the dominant non-Abelian correction and instead consider the sub-leading Abelian diagram (AB) we obtain the results labelled AB in Table 1. Based on the relative strength of the correction due to its colour factors we expect the results to be N_c^2 suppressed, and indeed this is the case. The mass of the pion is protected by chiral symmetry and so it receives negligible contributions from such corrections beyond rainbow-ladder. For the scalar, vector and a_1 axial-vector we see mass reductions of the order of $20-40$ MeV, while for the b_1 there is a slight repulsion of 3 MeV. This gives a strong indication that such Abelian corrections to rainbow-ladder are generally small and attractive.

Now, if we consider both the Abelian and non-Abelian corrections to the quark-gluon vertex together we might expect the effects on the meson bound-state to stack. That is, for the vector meson we would expect to see a ~ 120 repulsion from the NA correction, with a ~ 25 attraction from the AB correction, resulting in a bound-state mass of ~ 860. Instead, we see that the Abelian corrections are heavily suppressed by the non-Abelian ones for all meson channels, giving results that are almost identical to the those from the NA diagram alone. The conclusion is that naively adding and subtracting the results of different independent studies without performing the combined dynamical calculation can be misleading. It is thus reasonable, with the interaction model presented here, to ignore the Abelian diagram completely whenever we take the dominant non-Abelian diagram into account.

Finally, having demonstrated that the Abelian diagram is significantly suppressed in the meson spectrum, we consider the dominant non-Abelian corrections beyond rainbow-ladder with unquenching effects in the form of pion exchange (PI). This gives rise to the fifth row of Table 1. As expected, the inclusion of a pion exchange kernel is generally attractive, with an 80 MeV reduction of the vector meson mass with respect to the RL+NA result. This demonstrates, as suspected in [10, 16] the near cancellation of beyond-the-rainbow corrections with unquenching effects in this channel. Here, this is not an exact mechanism but the result of dynamical combinations of attractive and repulsive components of the quark-gluon vertex; a more accurate picture will be revealed when improved approximation schemes are employed that permit quantitative study.

5 Discussion and Outlook

We improved upon previous beyond-the-rainbow investigations, which focused upon pion unquenching corrections [17, 21, 22] and the dominant non-Abelian corrections to the quark-gluon vertex [16] by including the sub-leading Abelian corrections to the quark-gluon vertex.

The study we presented involves many technical developments in the calculation of the quark propagator and quark-gluon vertex at complex momenta. For highly non-trivial two-loop diagrams such as the crossed-ladder kernel, new and improved solution methods were required to render the calculation tractable. Despite these developments, the study remains qualitative and serves only as an exploratory study of the relevance of the different contributions beyond rainbow-ladder considered thus far.

Future investigations will focus upon including realistic input from the ghost and gluon sector of the theory. These are currently under investigation and the results will be reported elsewhere.

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